

COMMENTS ON ANTICIPATIONS

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Introduction

Anticipating a future theory of change, this paper comments on the phenomenon of anticipations in Dooyeweerd's systematic philosophy. The idea that reality itself or human experience of reality has some kind of a layered structure is put forward by several philosophers, but the insight that each aspect refers intimately to the others is uniquely Dooyeweerdian. It concerns an essential property of the structure of the modal aspects, each of which displays a 'meaning nucleus', expressive of its original meaning, besides retrocipations and anticipations, referring to the other aspects. Dooyeweerd assigns the aspects a linear temporal order, hence the distinction between retro- and anticipations. In fact, each aspect is considered to be an aspect of time as well as an aspect of being, a mode of human experience, and a principle of scientific explanation.¹

Both anticipations and retrocipations are crucial for the understanding of another Dooyeweerdian key concept, the 'opening up of reality' as a continuous natural and cultural process, or rather several such processes. By a 'cultural' process is meant any process in which human activity plays a leading part, including technology. For the development of the theory of the various opening processes, a theory of change is in need.

Finally, retrocipations and anticipations are very important for Dooyeweerd's theory of structural types. Recently I reviewed the

¹ The temporal order in a modal aspect is not the same as its meaning kernel. Therefore Dengerink makes a mistake by ascribing to me the view that the meaning kernel of the spatial aspect is not continuous extension, but simultaneity, see J.D. Dengerink (1996), book review of S. Griffioen, B.M. Balk (eds.) (1995), *Christian philosophy at the close of the twentieth century*, Kampen: Kok, in: *Phil.Ref.* 61, 196-205, p. 200. 'Simultaneity' is an original spatial expression of temporal order. I discussed and rejected Dengerink's view that both time and eternity should be considered separate modal aspects, see M.D. Stafleu (1988), 'Criteria for a law sphere (with special emphasis on the 'psychic' modal aspect)', *Phil.Ref.* 53, 171-186.

theory of 'thing-like' structures.² Strauss complains that he does not understand why I identify the foundational function of structural types with *retrocipations*.³ Now this is an important point. I did not invent it, I have no *a priori* or philosophical arguments for it, I did not find it in Dooyeweerd's *New critique* or any other authorized text, but I discovered it during my investigations. Claiming to be a realistic ontology, the Philosophy of the Cosmomic Idea should be empirical, corrigible and innovative.

Infinity

Anticipation means opening up and/or realization of possibilities in one modal aspect guided by laws in a later modal aspect. This savours of the age-old distinction of 'potential' and 'actual' being. For an example we shall discuss the fact that an irrational number such as $\sqrt{2}$ can be approximated by an infinite series of rational numbers.

Aristotle's acceptance of the potential infinite and his rejection of the actual infinite still echos in philosophy. According to Strauss, the potential infinite refers to a numerical *succession* of an endless series. The actual infinite refers to a set in which the members are *simultaneously* or *at once* present.⁴ The first example of a successively infinite set is constituted by the natural numbers themselves. They can be ordered in any way, for instance, 1, 3, 5, 2, 4, 6, ..., but their natural order is that of increasing magnitude. It is also the natural order for counting.

Let us apply Strauss's distinction to the rational and the real numbers. These are numbers deriving their meaning from being

² M.D. Stafleu (1994), 'De structuur der materie in de wijsbegeerte van de wetsidee', in: H.G. Geertsema et al. (eds.), *Herman Dooyeweerd 1894-1977, Breedte en actualiteit van zijn filosofie*, Kampen: Kok, 114-142.

³ D.F.M. Strauss (1995), 'The significance of Dooyeweerd's philosophy for the modern natural sciences', in: Griffioen, Balk (eds.), 127-138, p.134. This paper refers to M.D. Stafleu (1995), 'The cosmochronological idea in natural science', in: S.Griffioen, B.M. Balk (eds.), 93-111. See also: M.D. Stafleu (1989), *De verborgen structuur, Wijsgerige beschouwingen over natuurlijke structuren en hun samenhang*, Amsterdam: Buijten en Schipperheijn.

⁴ Strauss (1995), p.131-132. Strauss (p.132) says that 25 years ago I rejected the actual infinite. If he is right (I can only find stating that an infinite denumerable series has no actual limit, which I believe to be correct), I recant. See also D.F.M. Strauss (1996), 'Filosofie van de wiskunde' in: R. van Woudenberg (red.), *Kennis en werkelijkheid*, Amsterdam: Buijten en Schipperheijn, 143-176.

subject to numerical laws of addition, subtraction, multiplication, *etc.* The number of rationals between 0 and 1 is infinite. According to Strauss we find them by division, a 'successive' infinite process. In order to see this, put the rational numbers between 0 and 1 into the series $1/2, 1/3, 2/3, 1/4, 2/4, 3/4, 1/5, 2/5, \dots$. Clearly, this order is serial, and the rational numbers are 'denumerable', *i.e.*, they can be counted with the natural numbers serving as indices.

This order of the rational numbers is not 'natural', but the result of human intervention, it is artificial. The natural order of the rational numbers is the numerical order of increasing magnitude. Hence, the natural order of the above mentioned rational numbers is: $1/5 < 1/4 < 1/3 < 2/5 < 1/2=2/4 < 2/3 < 3/4$. However, in this order, the set of all rational numbers between 0 and 1 does not constitute a denumerable series, but a 'dense' one, see below. Hence, according to Strauss, whether the set of rational numbers between 0 and 1 is 'successive infinite' depends on how it is ordered. In its natural order it is not 'successively infinite', but it has the potential to become so, because of the possibility to create the above mentioned artificial ordering.

Infinite sets

Between every two rational numbers in the natural order it is always possible to find a third one (*e.g.*, half their sum). Therefore the set of rational numbers is called 'dense', there are no *extensive* holes, *i.e.*, holes of a magnitude larger than zero containing no rational numbers. Besides, the set of rational numbers between 0 and 1 is infinite, and because in the natural order it is not 'successively' infinite, it should be called 'simultaneously infinite', according to Strauss. Hence, whether we consider the set of rational numbers between 0 and 1 to be successively infinite or simultaneously infinite depends on their order. I have no objection, if I am allowed the comment that this 'simultaneity' is not first of all a property of the rational numbers, but of our treating them as a set. Indeed, speaking of a set, we assume that the elements of the set are simultaneously present, and if their number is infinite, we have an infinite number of elements simultaneously in the set. This may be called the spatial aspect of a set.

The concept of a set as applied in mathematics is a figment of the human mind. Perhaps nature abhors sets, but I do not, as long as the

theory of sets is not used as a logical foundation of mathematics. From the fact that the theory of sets struggles with peculiar paradoxes, it should not be concluded that sets are to be avoided. The paradoxes are mostly due to self-references and are logical rather than mathematical. For instance, the notorious ‘set of all sets that do not contain itself’ (which should contain itself if it does not) has little to do with mathematics. And the standard example of a paradox due to *circular* reference:

‘the next sentence is true’
 ‘the former sentence is false’

does not even refer to sets.

All this does not help us very much in understanding Strauss’s distinction of ‘successive’ and ‘simultaneous’ infinity. However, let us have a look at the real numbers.

Continuity (1)

Though being ‘dense’, *i.e.* having no extensive holes, the set of rational numbers between 0 and 1 is not ‘continuous’, because there are non-extensive holes between the rational numbers, due to the existence of numbers which are not ratios of natural numbers, like $\pi/4$ or $1/\sqrt{2}$. Referring to spatial continuity, it is possible to define the whole set of real numbers between 0 and 1 without leaving holes, as follows.

Consider a segment of a straight line. The set of spatial points between the two end points (*A* and *B*, say) is infinite. It is not merely dense, it is also continuous, *by stipulation*. Every set which members have a one-to-one correspondence to the members of a continuous set of spatial points is itself called ‘continuous’, *by definition*.⁵ Consider the set of all line segments having one common end point *A* besides having their second end points somewhere between *A* and *B*. Let us arbitrarily assume that the length of $AB = 1$. Now we define the set of all real numbers between 0 and 1 as the lengths of all line segments between *A* and *B*.⁶ This means that the set of real numbers between 0

⁵ M.D. Stafleu (1980), *Time and again, A systematic analysis of the foundations of physics*, Toronto: Wedge; Bloemfontein: Sacum, 32-42.

⁶ This is not an arbitrary definition, because the real numbers include the rational numbers. Hence, the number $1/2$ corresponds to the spatial point halfway between *A* and *B*. The fact that the rational numbers form a dense set warrants that the correspondence between the lengths of the line segments between *A* and *B* and the

and 1 is continuous, according to the definition. The set of real numbers between 0 and 1 derives its continuity from the continuity of the line segment AB . Whereas in the spatial aspect continuity is a primitive concept, in the numerical aspect it is derivative, anticipating the spatial aspect.

Now I put forward that the set of spatial points between A and B , and consequently, the set of real numbers between 0 and 1, each constitutes an 'at once infinity' in Strauss's sense. The spatial points are *by nature* 'simultaneously present' between A and B (not because they are members of a set). Moreover, the line segments between A and B do not arise from successive division of some line segment. By division one cannot arrive at the irrational numbers, as was already found by the Pythagoreans, to their dismay.⁷ As a matter of fact, there is no finite or infinite *numerical* procedure for finding all the real numbers *in succession*.

For this reason, I do not believe that the real numbers can be defined as limits of infinite series. Of course, it is not denied that many real numbers (including π) can be *calculated* (or rather approximated) as the limit of a prescribed series, but this cannot be done for all real numbers.

I conclude that 'continuity' can be attributed to the set of real numbers, as it cannot to the set of rational numbers. The definition of continuity refers to the set of spatial points in a line segment, which is called 'continuous' by stipulation. Hence, continuity as a property of real numbers refers to the spatial modal aspect.

Why do the irrational numbers anticipate the spatial and later aspects? It is not because of their belonging to any set, but because of their meaning. They are numbers because they are subject to numerical laws. But they determine quantities which are not purely numerical. The meaning of numbers like π or $\sqrt{2}$ refers to spatial (or kinematic, physical, ...) relations. It is a law that the ratio of the circumference and the diameter of all circles is π , and that the ratio of the diagonal and the side of any square is $\sqrt{2}$. These lengths are not defined as limits, even if we calculate their approximate values with

real numbers between 0 and 1 is unique. Only the assumption that $AB = 1$ is arbitrary. In a practical sense, it defines the length of the standard metre.

⁷ The assumption that all relations should be expressible as ratios of integral numbers formed an essential part of the Pythagorean philosophy. However, they discovered and proved that the ratio of the lengths of a diagonal and a side of a square cannot be rational.

the help of an infinite series or sum of rational numbers, like, for instance, $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \dots$

Actual measurements of spatial, kinematic or physical magnitudes like length, speed or energy always yield *rational* numbers, whereas theoretical considerations usually lead to the assumption that these magnitudes have *real* values, *i.e.*, they have a continuous range of possible values. Hence, the rational numbers are as numerical as the integers (being their ratio's), whereas real numbers (which include the rational numbers) serve as numerical anticipations on the later modal aspects.⁸

Sense or essence of meaning

Strauss would not agree with this reasoning, because in line with Dooyeweerd he considers 'continuity' to characterize the irreducible meaning nucleus of the spatial aspect. Some time ago, I suggested that in the process of opening up the anticipations of some modal aspect, its meaning is not only deepened but also relativized.⁹ In this way I tried to save the appearances: the fact that real numbers constitute a continuous set, anticipating the spatial aspect. But now I must confess that I no longer believe that the meaning nuclei of the first two modal aspects are adequately expressed by 'discrete number' and 'continuous extension', respectively. It just does not make sense.

If a set is discrete (or 'digital' as we would now perhaps say) it is countable, and countability is no doubt an important feature of certain sets. But there are sets which are not countable, such as the set of real numbers. Moreover, neither the rational nor the irrational numbers can be used for counting, but both are very useful in accounting for magnitudes and numerical relations. I suggest the meaning of the first modal aspect be given by numerical laws and their subjects.¹⁰

⁸ Magnitudes are also called 'variables', referring to their significance in a theory of change. For the analysis of change, differential and integral calculus is a very important instrument, anticipating the kinematic modal aspect.

⁹ M.D. Stafleu (1972), 'Metric and measurement in physics', *Phil.Ref.* 37, 42-57, p. 48. Also Stafleu (1980) 27, 42. According to Strauss (1995) 132, my 'alternative approach' is "uprooting" continuity, cancelling the qualifying role of the meaning nucleus of the numerical aspect with regard to its disclosed structure'.

¹⁰ H. Dooyeweerd (1953-1957), *A New Critique of Theoretical Thought*, 4 vols., Amsterdam: Paris, vol.2, 31: "... 'meaning' is nothing but the creaturely mode of being under the law, consisting exclusively in a religious relation of dependence on God ...".

Similarly, it does not make sense to say that the *meaning* of the spatial aspect would be *continuous extension*. In my view, the meaning of any modal aspect is given by the relation of its laws and its subjects, *i.e.*, everything created has dependent meaning, as a result of being subjected to law by its Creator.¹¹ The relation between modal laws and modal subjects is far too complex to be adequately given by a single word or expression.

However, there seems to be more at stake. I have the impression that for Strauss the 'meaning nucleus' is something like the *essence* or the *substance* of an aspect, in an Aristotelian sense. I believe this to be a far cry from Dooyeweerd's intentions.

A space is not merely continuous and extended, it is also dimensional and it has directions. In any space one counts a number of 'independent' dimensions. The mathematical idea of 'independence' is not easy to define, but it is intuitively comprehensible by realizing that in ordinary threedimensional space, any position is given by three independent measures, *e.g.*, length, width and height. These measures have to be given simultaneously, and are therefore subject to the spatial order of cosmic time. In the usual cases, the spatial points are given by 'vectors' (triples, in three-dimensional space) of real numbers, but it is very well possible to imagine a space determined by vectors of rational or integral numbers. Such a space is not continuous.¹²

The investigation of spatial dimensionality leads to the discovery of new types of numbers. Complex numbers were first contemplated when people discussed the meaning of taking the square root of a negative number. Their meaning as two-dimensional numerical vectors was established when the laws to which the complex numbers are subject were opened up. It was also discovered that such numbers only exist as one-dimensional, two-dimensional and four-dimensional vectors, the real and complex numbers besides the quaternions. It can be proved from numerical laws that no other possibilities exist, and therefore their definition is not arbitrary. Hence, the meaning of any kind of number is determined by numerical laws only, as opened up by the later modal aspects.

¹¹ Stafleu (1980) 25; (1995) 93-94.

¹² Structures based on non-continuous spaces are known to lead to so-called 'fractals'.

Opening up and relativizing

The idea that by developing the anticipations of a modal aspect one also relativizes its meaning is illustrated by the theory of relativity. In 1905 A. Einstein discovered that the order of simultaneity is no longer absolutely valid if one considers spatially remote events from the viewpoint of differently moving frames of reference.¹³ This leads to the conclusion that the spatial order of simultaneity is relativized if the spatial aspect is anticipating the kinematic one.

A triangle's sides function as boundaries in a two-dimensional space, they separate the inner space from the outer space. The inner space has a certain magnitude, the triangle's area, and according to medieval views, influenced by Aristotle, the boundaries also determine the 'space', *i.e.*, the place of the triangle. This view gradually developed into the idea that the triangle is 'somewhere in space'.

The Newtonian idea of 'space' (not by accident, this is a noun) also seems to imply that space is a thing, a container, something we live in. But if we speak of the 'spatial aspect' we can never mean a 'something', a unique being, a concrete entity. A modal aspect is a mode of temporal being and of understanding, a principle of explanation. It is also one way of relating things or events to each other. One of the most important parts of a modal aspect is its characteristic 'subject-subject-relation'. Hence, the concept of a co-ordinate system or reference system comes closer to what we should understand by a 'space' than the Newtonian concept of a container. The theory of relativity discusses the effects of mutually moving systems of reference. Therefore, we could say that it considers relatively moving spaces. In each space the usual order of simultaneity is maintained. Only if we compare the orders of simultaneity with respect to spaces which move relatively to each other, we get different results.

¹³ These systems are not merely spatial, they also include clocks. If according to one system of reference an event is simultaneous with another one, in another system the two events may occur one after the other, excluding the possibility that one of them is the cause of the other.

Hearing and seeing

Sound is a linear process, music beats serial time. Sound is a combination of waves, and each wave is characterized by its frequency or pitch, its amplitude or volume, and its phase. We hear the same sound with both ears, and for every wave, we experience a slightly different amplitude and a different phase. By comparing them we are able to determine from which direction the sound is coming. From our experience we can estimate the distance of the source. Hence, hearing allows us to orient ourselves in our environment. Estimating his distance to a car, a cyclist uses his ears nearly as much as his eyes.

Seeing is two-dimensional in principle, and by some tricks we are even able to see three dimensions. Both in hearing and in seeing, not only space but also motion and physical processes are involved. Moving objects are better visible than stationary ones, and if an object does not move, we increase its visibility by moving our head or our eyes.

Seeing is natural to mankind, most mammals, birds, squids and insects. Like hearing, seeing is an interesting and complicated process. It involves physics, physiology, psychology and logic, and even more, hence it is characterized by anticipations.

As a physical process, it was studied by Greek, Roman, Arab and Western scientists. Abu Ali al-Hasan ibn al-Hasan ibn al-Haytam (circa 965-1039), in the West better known as Alhazen, laid the foundations on which Johannes Kepler in 1604 built the theory which is still accepted.¹⁴ If we look at a tree, an image of the tree is formed at the retina, the backside of our eye. Alhazen showed that the image is not formed at the object and transported to the eye, but is formed inside the eye, after light from the object has travelled to the pupil. More than 300 years after the eye-piece came into use for reading, Kepler proved that the human eye acts like a lens. (Technology often anticipates scientific discovery). Mammals, birds and squids make use of a lens, insects do not. The retina consists of cells sensitive to light. For mammals, these cells lay at the back of the retina, for squids at the front. Some animals are able to discern colours, others do not. The physiological process starts at the retina, when signals are transmitted

¹⁴ D.C. Lindberg (1976), *Theories of vision from Al-Kindi to Kepler*, Chicago: University of Chicago Press.

to the brain. Kepler emphasized that he could explain the physical aspect of seeing, but not the physiological and psychic ones. In the brain the image invokes emotions, leading to actions. The actions may be programmed by inheritance, or are based on earlier experience, or are performed after some thinking. For instance, when looking at an oncoming car we estimate its distance and speed, before crossing the road. Cats, dogs and birds do that, too. The image may be stored in the memory, short-term or long-term, and compared with earlier images. Hence, as a temporal sequence, the physical process anticipates the physiological one, which in turn anticipates the psychic aspect, which anticipates the logical aspect of seeing. Sound and light may bear signals, which can be understood or interpreted.

The physical process involves the formation of an image by a lens. Although we call it a physical process, as a natural process it only occurs in animals. Hence, it is not purely physical, it can only be understood as anticipating the psychic functioning of seeing. Moreover, the physical process is guided by physiological processes. For instance, guided by the brain the focussing of the lens is adopted to the distance of the object which we want to observe.

Plants too are sensitive to light. With a few exceptions, plants derive their energy from photosynthesis. Still, nobody would say that a plant can see. In plants the biotic process is the end.

We only speak of seeing if a specialized living organ is involved, which may be as complicated as the human eye, or as simple as a single light sensitive part of a monocellular organism. Such an organ is easily recognized having a biotic structure with a psychic function. But the functioning of these organs cannot be understood by an analysis of their 'thing-like' structure, because they are not characterized by their physical or biotic structure, but by their functioning in the process of seeing.

We speak of seeing only if it leads to psychic actions. In seeing the physiological or biotic process following the physical one is not purely biotic, but anticipates the psychic aspect. If this only invokes emotions, the process as a whole is qualified by the psychic aspect, but whatever we see may lead to logical processing and to experiences and acts which are qualified by one of the later modal aspects, such as the aesthetic one.

Sometimes the eyes deceive us in seeing things that do not exist apart from seeing it, such as the rainbow. The rainbow is an image on

the retina formed by the lens (therefore it can also be photographed)—but it is not an image *of* something.

Continuity (2)

There is a remarkable continuity in the processes of seeing and hearing. Analysing how an artist conceives of a painting and puts it on linen, we discern easily the various modal aspects and their relations, retro- and anticipations. But it is a single process. This continuity is also found in the structure of things, but even more so in processes. We have to be aware that the mutual irreducibility of the modal aspects and their inherent discontinuity is a necessary condition for understanding our world, but is not a basic property of the structures of individuality. In particular processes show a continuity which transcends the limits of the modal aspects.

For example, consider the growth of a human being starting from a fertilized egg via a new born baby to an adult person. The fertilized egg is only a biotic subject, although it has the potential to become a human being. Only gradually it opens up its psychic, logical, formative, and other functions, and nobody can say at what moment the embryo becomes a feeling subject (having emotions), or a thinking subject, *etc.* The crossing of the boundaries between the various aspects is a continuous process.

Processes

According to Dooyeweerdian orthodoxy, a structural whole should have a primary qualifying function, a secondary founding function, and a tertiary destination function.¹⁵ ‘Function’ refers to a modal aspect. Hence, the foundation function is expressed by some retrocipation of the qualifying aspect, and the destination function to some anticipation. A biochemical process, for instance, would be physically qualified, kinematically founded, and finds its ultimate meaning in the biotic aspect.

However, Dooyeweerd’s typology concerns thing-like structures. Moreover, besides the qualification function he stressed in particular the foundation function, and the tertiary destination function was often identified with the primary qualification function. His typology

¹⁵ Stafleu (1994).

concerns first of all the structure of individual subjects, whereas in processes always several subjects are involved. The structure of a process is partly determined by the structures of the subjects involved, but cannot be derived from them. In particular a process leading to the formation of a new structural whole, such as occurs in a chemical reaction, has its own structure, which is not adequately described by Dooyeweerd's theory of structural types.

In a process subject-subject relations get a structure. If the product of a process is a structured thing, the process and all subjects involved in the process anticipates that thing. For instance, in a chemical process in which molecules *A* and *B* are connected to form a new molecule *C*, both the molecules *A* and *B* as well as the process anticipate the structure of *C*.

In such processes besides the structures of *A*, *B* and *C*, also the energy and the entropy play important parts. Both energy and entropy (and related properties like free energy) are modal concepts, and the laws concerning them are modal laws. The second law of thermodynamics was developed about 1850 by R. Clausius and W. Thomson, the concept of entropy was introduced by Clausius about 15 years later. It was developed along two lines, one mathematical, the other chemical. In a mathematical way, L.E. Boltzmann related entropy to probability in the context of statistical mechanics. For the theory of processes the chemical concept of entropy is more relevant. It is also useful to describe equilibrium situations.

In biological processes information becomes relevant, another modal concept. In all biotic processes the transfer of information, and in psychic processes the handling or processing of information is highly significant.

Possibilities, probabilities

Anticipation means opening up and realization of possibilities in one modal aspect guided by laws in a later modal aspect. In a mathematical way, possibilities can sometimes be measured by probabilities.

Playing dice involves a physical and kinematic process, but assuming that the die is homogeneous, the calculation of the probable outcome of a throw is based on the geometry of the die. Even if the number of possibilities is infinite, probabilities can be calculated by a

suitable division of the domain of possible outcomes. In this way, J.C. Maxwell was able to calculate the distribution of molecular speeds in a gas at a given temperature.¹⁶ In order to arrive at his distribution law, he had to make some arbitrary assumptions about the way the molecules interact with each other. He was able to derive some unexpected and later verified results, but at the end of his paper he observed that his theory could not account for the observed specific heat of the then known gases.

Quantumphysics discovered that the motion of a particle like an electron or a photon has a wavelike character, which should be interpreted as providing possible paths and probable destinations of the moving particle. This is an example of motion anticipating physical interaction. Since the introduction of quantum physics it is accepted that many processes are not determined by mechanical laws, but display a certain amount of intrinsic stochastic behaviour.

A fundamental problem for any theory of change concerns the transition of a possibility to its realization. When throwing a die, at what moment becomes the probability to get a six a certainty? With respect to games of chance this problem is largely academic, but in quantum physics it is a major and still unsolved problem.

Numerical structures

Because the standard Dooyeweerdian typology mostly concerns structures having a foundation function, we do not find numerically qualified structures in this way. In fact, Dooyeweerd supposed that the physical aspect is the first to qualify a structure of individuality.¹⁷ Elsewhere I argued that at least there are spatially and kinematically qualified structures.¹⁸ Now I wish to emphasize that numerical structures *anticipating* the later aspects are not difficult to find. I shall mention only one example, the structures called 'groups'.¹⁹

¹⁶ J.C. Maxwell (1860), 'Illustrations of the dynamical theory of gases', *The scientific papers of James Clerk Maxwell* (1890, W.D.Niven, ed.), 2 vols. bound in one, New York 1965: Dover, vol.1, 377-409.

¹⁷ Dooyeweerd (1953-57) vol.2, 425, vol.3, 79, 99.

¹⁸ M.D. Stafleu (1985), 'Spatial things and kinematic events (On the reality of mathematically qualified structures of individuality)', *Phil.Ref.* 50: 9-20.

¹⁹ Stafleu (1980) 35ff.

In an abstract sense, a group is a set of elements such that each pair of elements generates another one. If A and B are elements, then the combination AB is also an element of the group. For instance, if A and B are numbers, AB may stand for addition ($A+B$) or multiplication ($A*B$).

The group contains an identity element (I), such that for each other element A , $AI = IA = A$. For addition, the number 0 is the identity element, for $A+0 = 0+A = A$. For multiplication, the number 1 is the identity element, for $A*I = I*A = A$.

Each element A has an inverse A' , such that $AA' = A'A = I$. For addition, $-A$ is the inverse of A , for multiplication, it is $1/A$.²⁰

Consider one simple finite group, consisting of 'operations' in a plane: the clockwise rotation through 90° , through 180° , and through 270° . The identity element is rotation through 0° , and the inverse elements are anti-clockwise rotations through -90° , -180° and -270° . It is clear that the latter three are identical to the former three in reversed order. Hence the group contains four elements. The group describes the rotation symmetry of a square.²¹

The described group is not a square itself. It is a mathematical structure anticipating a spatial 'thing', but it is purely numerical. One could object that the group is defined with the help of the expression 'rotation' which is clearly spatial. However, the definition of the group is not in need of this specification. It is sufficient to define the group as having four elements, I, A, B, C , with $AA = B = B'$, $A' = AB = C$, and $BC = A$. But its meaning is deepened if we see that this group anticipates the symmetry of a square.

The theory of groups is quite complicated, and has been very fruitful in solving many problems in atomic, sub-atomic, molecular and solid state physics and chemistry. Hence a group has both a primary and a tertiary structure, but not a secondary one: it lacks a founding function. Because of its anticipatory character, the theory of groups will play an important part in any theory of change.

²⁰ Also, for a group $(AB)C = A(BC)$. The reader who is not familiar with groups is invited to check that the set of integral numbers is a group under addition, but not under multiplication, and that the set of positive rational numbers is a group under multiplication, but not under addition. Remember that division by 0 is not allowed. Groups may be finite or infinite.

²¹ A square is also symmetrical with respect to mirroring with respect to a diagonal etc. We can extend the group of rotations to include mirrors, in order to account for the full symmetry of a square.

Repeatedly, I have been asked why I do not consider pure numbers to have a mathematical structure, and why there is no 'kingdom' of numbers.²² In our philosophy, we distinguish between 'modal laws' and 'structural laws'. Structural laws refer to a restricted class of typical subjects, having an *internal* structure which distinguishes them from other subjects. Squares are different from circles, but both are spatial subjects. Also modal laws refer to subjects, which I have baptized 'abstract modal subjects', which, by the way, are no less 'real' than things or events. Modal subjects are discovered as soon as people start to abstract the modal aspects from reality. This is not necessarily a scientific procedure, for children are able to learn the numbers and the laws of addition and multiplication without becoming mathematicians. Yet, the numbers are a product of human cultural activity. The possibility (or even necessity) to develop numbers is based on the laws laid down for the creation, therefore our number system is not fully arbitrary. (Clearly, the fact that we use a system based on factors of ten is culturally determined.)

It may be a matter of taste (certainly not mine) to ascribe a structure to the prime numbers. They certainly have a property which distinguishes them from other numbers, but so have the even numbers, the odd numbers, the triples, etc. If having a single property would lead to the concept of a structure, then we should assume that all red things also have a common structure. The point is that the properties of these numbers can be derived from the modal laws only, we do not need additional typical laws. The rational numbers and the real numbers, too, are not defined by some typical law, but are discovered by analysis of the modal laws for the numerical and spatial aspects.

We may speak of a structure if it has more properties than are needed for its definition, and if there is a specific law for them that is different from the general modal laws. Thus, after defining a square as a spatial two-dimensional figure having four equal sides and equal angles, we discover that it has the rotation symmetry described by the above mentioned group (having the character of a typical *law*), we may speak of a typical structure. But even this criterion is probably not watertight.

²² J. Stellingwerff (1990), Book review of Stafleu (1989), *Phil.Ref.* 55, 94-95.

Against reductionism

Reductionism is a collective noun for a variety of philosophies that seek ultimate explanations in what we call the retrocipatory spheres of the modal aspects. (In general, the mutual irreducibility of the modal aspects is denied.) Examples are mechanism (reduction to the kinematic aspect), naturalism (reduction to physics), evolutionism (reduction to the biotic aspect). For instance, to reduce the moral aspect of human experience to biotic needs is nearly a commonplace.

Reductionist philosophers lack the insight into the distinction between animals and human beings. The functioning of animals in the post-psychic aspects is retrocipatory, determined by their biotic and psychic needs, but human activity is characteristically anticipatory.²³

It has little sense to fight naturalism or evolutionism only on immanent or transcendental grounds.²⁴ In order to deliver an effective criticism, one has to show the fruits of an alternative approach to evolution and anthropology. Hence, the study of anticipations is crucial for the rebuttal of reductionist philosophies.

²³ For the relevance of anticipations for anthropology, in particular with respect to the distinction of animals and human beings, see M.D. Stafleu (1991), 'Being human in the cosmos', *Phil.Ref.* 56: 101-131; Stafleu (1995).

²⁴ For the distinction between immanent, transcendent, and transcendental critique and their effectiveness, see M.D. Stafleu (1987), *Theories at work, On the structure and functioning of theories in science, in particular during the Copernican revolution*, Lanham, New York, London: University Press of America, 205-210.